Prairie Discrete Mathematics Workshop

Brandon University

June 12 to June 15, 2018

Conference Program

Conference Sponsors

The Pacific Institute for the Mathematical Sciences (PIMS)
Research Manitoba
The Fields Institute
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The Canadian Mathematical Society (CMS) Student Committee

Invited Speakers

- **Richard Brewster**
  Thompson Rivers University
  Reconfiguration problems
- **Ada Chan**
  York University
  Fractional revival in continuous-time quantum walk
- **Rob Craigen**
  University of Manitoba
  Synthetic Orthogonality Theory: Boldly going where math has not gone before
- **Shonda Gosselin**
  University of Winnipeg
  The metric dimension of circulant graphs and Cayley hypergraphs
- **Gary MacGillivray**
  University of Victoria
  Frugal colourings and homomorphisms
- **Karen Meagher**
  University of Regina
  Spectrums that can be realized by an adjacency matrix of a graph

Conference Organizers

- Shahla Nasserasr, Brandon University
- Sarah Plosker, Brandon University

Schedule

	Tues 12	Wed 13	Thurs 14	Fri 15
8:00–8:55		Breakfast	Breakfast	Breakfast
9:00–9:50		Chan	MacGillivray	Craigen
10:00–10:25		coffee	coffee	coffee
10:30–10:55		Zhan	Duffy	Vasudevan
11:00–11:25		Herman	Shirazi	Erzurumluoğlu
11:30–11:55		Singh	Vander Meulen	Depart
12:00		Photo, lunch break (12-1:50)	lunch break (12-1:50)	
1:30	Registration (1:30-2:25)			
2:00–2:25		open problems	Deaett	
2:30–3:20	Gosselin		Meagher	
3:30–3:55	Butler	coffee	coffee	
4:00–4:25	coffee	open problems	Garnett	
4:30–4:55	Monfared		Brewster	
5:00–5:25	Kroeker			
5:30–5:55	Mol	posters & pizza	Quinlan	
6:00–			BBQ	

List of Speakers and Abstracts

Title: Minimizing the largest mean first passage time of a Markov chain: the influence of directed graphs
Speaker: Jane Breen, University of Manitoba
Co-author: Steve Kirkland, University of Manitoba
Abstract: For a Markov chain described by an irreducible stochastic matrix T of order n, the mean first passage time $m_{i,j}$ measures the expected time for the Markov chain to reach state j given that the system begins in state i, thus quantifying the short-term behaviour of the chain. We give a lower bound for the maximum mean first passage time in terms of the stationary distribution vector of T. We also discuss the characterisation of the directed graphs D for which any stochastic matrix T respecting this directed graph attains equality in the lower bound, thus producing a class of Markov chains with optimal short-term behaviour.

Title: Reconfiguration Problems
Speaker: Rick Brewster, Thompson Rivers University
Co-authors: JaeBaek Lee and Mark Siggers, Kyungpook National University, Mathematics Department, Daegu South Korea.
Abstract: In a classic combinatorial decision problem one is given an instance and a question, for example a graph G and the question "Does G admit a k-colouring?". A standard problem is then to classify a family of such problems into those that can be solved in polynomial time and those that are NP-complete. In *combinatorial reconfiguration problems* an instance includes two solutions to some combinatorial problem, for example a graph G and two k-colourings f and g. The question is then can the first solution be *reconfigured* into the second through a sequence of allowed (small) transformations? For example, can the k-colouring f be reconfigured into g by recolouring one vertex at a time so that all intermediate colourings are proper k-colourings? Here the typical dichotomy is polynomial versus PSPACE-complete.

In this talk we will examine the general reconfiguration framework. Some key PSPACE-completeness reductions, including the Nondeterministic Constraint Logic of Hearn and Domaine, will be covered. Our main focus will be a reconfiguration problems for graph colourings and graph homomorphisms. New results on reconfiguring homomorphisms to reflexive cycles (joint with Lee, and Siggers) will be presented.

Title: A forest building process on graphs
Speaker: Steve Butler, Iowa State University
Co-authors: Zhanar Berikkyzy (UC Riverside), Jay Cummings (Sacramento State University), Misa Hamanaka (Iowa State University), Maried Hardt (Iowa State University), Kristin Heysse (Macalester College), Paul Horn (University of Denver), Ruth Luo (University of Illinois, Urbana-Champaign), Brent Moran (Freie Universiät Berlin)
Abstract: Consider the following process on a simple graph without isolated vertices: order the edges randomly and keep an edge if and only if it contains a vertex which is not contained in some preceding edge. The resulting set of edges forms a spanning forest of the graph.

The probability of obtaining k components in this process for some families of graphs is determined as well as a formula for the expected number of components in any graph. A generic recurrence and some additional basic properties are discussed.

Title: Fractional revival in continuous-time quantum walk
Speaker: Ada Chan, York University
Abstract: The continuous-time quantum walk on a graph X is given by the unitary operator e^{-itA}, where A is the adjacency matrix of X. The graph X admits fractional revival from u to v at time τ if

$$e^{-i\tau A} = \alpha e_u + \beta e_v,$$

for some $\alpha, \beta \in \mathbb{C}$. Here e_u and e_v denote the characteristic vectors of vertices u and v, respectively.

Perfect state transfer from u to v and periodicity at u are two special cases of fractional revival with $\alpha = 0$ and $\beta = 0$, respectively. These two properties have been extensively studied in the past 15 years.

In this talk, we focus on graphs admitting fractional revival where both α and β are non-zero. We give an overview of what we know about these graphs.

Title: Synthetic Orthogonality Theory: Boldly going where math has not gone before
Speaker: Robert Craigen, University of Manitoba
Abstract: A decade ago de Launey and Flannery mapped the relatively new mathematical world of Algebraic Design Theory in a book by the same name. Being asked to write the AMS Math Review for the book gave me the opportunity to relive this adventure vicariously. We debarked at the outpost planet Combinatorial Orthogonality, chartering a freighter to a destination in the ambient ring. I may tell that story another time; what interests me presently is the surprisingly unexplored terrain around the outpost. On a return journey I skipped the freighter and set out to explore Combinatorial Orthogonality without a compass or map, armed only with my wits, and not taking as my end the establishment of a bridgehead for algebra. I found many pretty stones; this talk is a travelogue and show-and-tell.

Title: Canny Edge Detection Algorithm: Vectors Orthogonal to Edges Applied to Cosmology and Astrophysics
Speaker: Rebecca Danos, University of Winnipeg
Abstract: In this poster we present an advanced and optimized implementation of the Canny algorithm and apply it to finding vectors orthogonal to edges in various cosmology and astrophysics applications. A prior implementation first studied the Canny algorithm to search for cosmic strings in (Amsel, Berger, and Brandenberger 2008). We modified the algorithm to optimize its performance and applied it to place constraints for cosmic string and superstring signatures in (Danos and Brandenberger 2010a, 2010b), as a novel and alternative method to place contraints on non-Gaussianities in the CMB in (Danos, Frey, and Wang in 2012) and to suggest an automated means to search for supernova remnants with Safi-Harb.

Title: A minimum rank problem for matroids
Speaker: Louis Deaett, Quinnipiac University
Abstract: Suppose we have a combinatorial description of some matrix, such as a digraph or pattern specifying which of its entries are nonzero. What bounds can we put on the rank of the matrix? We consider a purely combinatorial version of the problem, where we consider the rank function of a matroid. We also look at how the theory of matroid representability can inform our investigation of the original matrix-theoretic problem.

Title: Surveying the Landscape of Colouring Parameters for Oriented Graphs
Speaker: Chris Duffy, University of Saskatchewan
Abstract: When extended to directed graphs, the homomorphism interpretation of graph colouring gives a definition of proper colouring for directed graphs that takes into account the direction of the arcs. In this talk I will survey the landscape of colouring parameters that arise from this definition, review major results and techniques, and discuss future directions and conjectures in this area.

Title: Hamiltonian Properties of 2-Block-Intersection Graphs of Twofold Triple Systems
Speaker: Aras Erzurumluoğlu, University of Ottawa
Co-author: David Pike, Memorial University of Newfoundland
Abstract: A balanced incomplete block design (BIBD(v, k, λ)) (V, B) is a combinatorial design in which (i) $|V| = v$, (ii) for each block $B \in \mathcal{B}$, $|B| = k$, and (iii) each 2-subset of V occurs in precisely λ blocks of $\mathcal{B}$. A BIBD(v,3,2) is a twofold triple system (TTS(v)).

Given a combinatorial design $\mathcal{D}$ with block set $\mathcal{B}$, the i-block-intersection graph (i-BIG) of $\mathcal{D}$ is the graph having $\mathcal{B}$ as its vertex set, where two vertices $B_1 \in \mathcal{B}$ and $B_2 \in \mathcal{B}$ are adjacent if and only if $|B_1 \cap B_2| = i$.

Recently we have settled the spectrum of TTSs with connected non-Hamiltonian 2-BIGs, as well as the spectrum of TTSs with Hamiltonian 2-BIGs. In this talk I will present some of these results together with the techniques that were used to obtain them.

Title: Algebraic Conditions that preclude SAPpiness
Speaker: Colin Garnett, Black Hills State University
Co-author: Louis Deaett, Quinnipiac University
Abstract: This talk focuses on several algebraic conditions on the coefficients of the characteristic polynomial that can be exploited to show that a pattern is not spectrally arbitrary over any field. Using Sage we were able to show that no zero-nonzero pattern with $2n - 1$ nonzero entries will be spectrally arbitrary over C where $n \leq 6$. When $n = 7$ we find two zero-nonzero patterns that do not satisfy our algebraic conditions precluding them from being spectrally arbitrary. This talk will discuss the implications of our test and some details of using sage to search for SAPs.

Title: The metric dimension of circulant graphs and Cayley hypergraphs
Speaker: Shonda Gosselin, niversity of Winnipeg
Co-authors: Adam Borchert (University of Cambridge), Kevin Chau (University of Winnipeg)
Abstract: A pair of vertices x and y in a graph (or hypergraph) G are said to be resolved by a vertex w if the distance from x to w is not equal to the distance from y to w. We say that G is resolved by a subset of its vertices W if every pair of vertices in G is resolved by some vertex in W. The minimum cardinality of a resolving set for G is called the *metric dimension* of G. The problem of determining the metric dimension of a graph is known to be NP-hard (Khuller et al 1994). The metric dimension of a graph has applications in network discovery and verification, combinatorial optimization, chemistry, and many other areas, and consequently this graph parameter has received a great deal of attention from researchers, the main goal being to determine the metric dimension of certain classes of graphs or to achieve asymptotic bounds. In particular, there is great interest in finding classes of graphs whose metric dimension does not grow with the number of vertices. Such graphs are said to have bounded metric dimension. In this talk, we bound the metric dimension of Cayley hypergraphs on finite Abelian groups with the canonical set of generators, and we show that the metric dimension of these hypergraphs is equal to the metric dimension of a Cartesian product of a circulant graphs of the form $C_n(1, 2, \ldots, t) = Cay(\mathbb{Z}_n : \{\pm 1, \pm 2, \ldots, \pm t\})$. These circulant graphs have bounded metric dimension (Grigorious et al 2014). In fact, their metric dimension is determined by the congruence class of n modulo $2t$. We present some background on the problem and some new results. We also bound the metric dimension of Cartesian products of these circulants, which yields bounds on the metric dimension of the corresponding Cayley hypergraphs.

Title: From Association Schemes to Hopf algebras
Speaker: Allen Herman, University of Regina
Co-author: Gurmail Singh, University of Regina
Abstract: If one starts with the adjacency algebra of an association scheme, one has the associative algebra part of a Hopf algebra whose antipode is the transpose map. But one needs to define a suitable counit and coassociative comultiplication for the adjacency algebra to be extendible to an actual Hopf algebra. Can this always be done? I will report on some preliminary results on this question.

Title: Fractional Calculus
Speakers: Thomas Humphries, Hunter Plowman, Brandon University
Abstract: Fractional calculus is the theory of integrals and derivatives of arbitrary order, which unifies and generalizes the integer-order differentiation and n-fold integration. We will present several simple fractional derivatives and integrals, such as $\frac{d^{\frac{1}{2}}}{dx^{\frac{1}{2}}}x$, or $\frac{d^{\frac{1}{2}}}{dx^{\frac{1}{2}}}\sin x$, along with a background introduction.

Title: On the mean connected induced subgraph order of cographs
Speaker: Matthew Kroeker, University of Winnipeg
Co-authors: Lucas Mol and Ortrud Oellermann (University of Winnipeg)
Abstract: The *mean connected induced subgraph order* of a graph G, denoted M_G, is the sum of the orders of all connected induced subgraphs of G divided by the number of such subgraphs. A *cograph* is a graph with no induced path on 4 vertices. Among all connected cographs of a fixed order n, we show that the *star* $K_{1,n-1}$ has maximum mean connected induced subgraph order and the *n-skillet* $S_n = K_1 + (K_1 \cup K_{n-2})$ has minimum mean connected induced subgraph order. The *density* of a graph G of order n is given by M_G/n. We use the established bounds on M_G to conclude that the density of connected cographs is asymptotically $1/2$.

Title: Frugal colourings and homomorphisms
Speaker: Gary MacGillivray, University of Victoria
Co-author: Stefan Bard, University of Victoria
Abstract: A k-colouring of a graph G is t-frugal if, for every vertex x of G, at most t vertices in the neighbourhood of x have the same colour. We will survey what is known about frugal colourings, and in doing so pay particular attention to the following topics: the complexity of deciding whether a given graph has a t-frugal k-colouring, bounds on the number of colours required to t-frugally colour a graph of given maximum degree, frugal colourings of graphs in some special graph classes, and t-frugal homomorphisms.

Title: Spectrums that can be Realized by an Adjacency Matrix of a Graph
Speaker: Karen Meagher, University of Regina
Abstract: For every graph there is an associated a family of matrices that represent the graph. The rows and columns of these matrices are indexed by the vertices of the graph, and the entries are non-zero if the corresponding vertices are adjacent and zero if their are not adjacent. For a given graph G, this family is denoted by $S(G)$.

One large and complicated problem in discrete math is to determine properties of $S(G)$ for a given graph G. In this talk I will focus on spectral properties for the matrices in this family.

First I will consider the minimum number of distinct eigenvalues that a matrix in $S(G)$ can have, given structural properties of the graph. Next I will show that the set of graphs G that have a matrix in $S(G)$ with only two distinct eigenvalues (which is the smallest possible, without being trivial) appears (at least at present) to be very difficult to characterize. To make this problem more tractable, I will consider properties of graphs that have a matrix in $S(G)$ that have only two distinct eigenvalues with one eigenvalue with a fixed multiplicity of k. When $k = 1$, it is easy to characterize these graphs. For $k = 2$ the characterization is a little more difficult but known. But for $k \geq 3$ no characterization is know. I will give a summary of what is currently know and open problems that I think are feasible.

Title: Quantum probability measures
Speaker: Darian Mclaren, Brandon University
Co-authors: Sarah Plosker (Brandon University), Christopher Ramsey (University of Manitoba & Brandon University)
Abstract: The notion of a classical probability measure is generalized in quantum mechanics to a quantum probability measure that is matrix- or, more generally, operator-valued. Our goal is to study these measures, building up the operator theoretic background, with focus on examples.

Title: The Circular Repetition Threshold for Small Alphabets
Speaker: Lucas Mol, University of Winnipeg
Co-authors: James D. Currie and Narad Rampersad
Abstract: A word is a finite or infinite string of letters taken from some finite alphabet. A word w is called a β-power if we can write $w = x^\beta$ for some word x. In this case, β is called the *exponent* of w. For example, the English word $\texttt{onion} = (\texttt{oni})^{5/3}$ is a $\frac{5}{3}$-power. If we treat `onion` as a *circular word* by linking up the ends, then it also contains the 2-power $\texttt{onon} = (\texttt{on})^2$ as a factor. A (circular) word is called β-free if it has no factors of exponent greater than or equal to β. The *strong circular repetition threshold* for circular words, denoted $\mathrm{CRT}(k)$, is the infimum of the set of all β such that there are β-free circular words of every length on k letters. We find the last two unknown values of the strong circular repetition threshold, namely $\mathrm{CRT}(4) = \frac{3}{2}$ and $\mathrm{CRT}(5) = \frac{4}{3}$, confirming a conjecture of Gorbunova.

Title: Graph partitioning problems arising in neuroscience
Speaker: Keivan Hassani Monfared, University of Calgary
Co-authors: Kris Vasudevan (University of Calgary), Jordan Farrell (Stanford University), and Gordon Campbell Teskey (Hotchkiss Brain Institute, University of Calgary)
Abstract: Functional connectivity network of the brain is a weighted graph with positive and negative weights. The weight of an edge $\{i, j\}$ is the correlation between the signals recorded from vertex i and vertex j in the brain. One of the common challenges in computational neuroscience is to meaningfully partition the vertices of the graph into an unspecified number of clusters so that the vertices in each cluster are most similar to each other and least similar to the vertices in other partitions. In this talk I will introduce some classic and some new (heuristic, spectral, and optimization) approaches to clustering graphs, their generalization to signed graphs, and the challenges that these method present.

Title: Minimally primitive and exponent-critical graphs
Speaker: Rachel Quinlan, National University of Ireland Galway
Abstract: A directed graph Γ is *primitive* if there is a positive integer k for which there exists a walk of length k in Γ from u to v for every pair (u, v) of vertices in Γ; the least such k is called the *exponent* of Γ. A primitive graph is *minimally primitive* if the deletion of any arc leaves an imprimitive graph, and *exponent-critical* if deletion of any arc either increases the exponent or leaves an imprimitive graph. We will discuss some features of graphs with these properties, including their behaviour under the graph Kronecker product and the range of possible exponents of mimimally primitive graphs.

Title: Review of Some Nonexistence Results for Strong External Difference Families
Speaker: Mahsa N. Shirazi, University of Regina
Abstract: In 2015, Paterson and Stinson introduced a certain type of design, named Strong External Difference Families (SEDFs) and showed that SEDFs have an important application in Algebraic Manipulation Detection (AMD) codes. To be more Specific, The optimal strong AMD can be obtained from SEDFs. But in 2017, Martin and Stinson focused on SEDFs as an interesting mathematical problem in its own right. In their paper they used some results from representation theory to disprove the existence of SEDFs when the group is finite abelian of prime order. We will consider the possibility of extending the result to other groups and if they can be applied to other designs.

Title: Non-homogenous Fourier matrices
Speaker: Gurmail Singh, University of Regina
Abstract: A Fourier matrix S is a unitary symmetric matrix with positive entries of the first column that produces integral structure constants using Verlinde's formula $N_{ijk} = \sum_l S_{li} S_{lj} \bar{S}_{lk} S_{l0}^{-1}$. In this talk, we will classify non-homogeneous Fourier matrices under certain conditions up to rank 5 by using the properties of C-algebras arising from Fourier matrices.

Title: Well-Covered and Vertex Decomposable Graphs
Speaker: Kevin Vander Meulen, Redeemer University College
Co-authors: Jonathan Baker (Brock) and Adam Van Tuyl (McMaster)
Abstract: A graph G is well-covered if every maximal independent set of G has the same cardinality. We explore a subset of well-covered graphs that are vertex decomposable. There are some known constructions of well-covered vertex decomposable graphs; for each of these, we show that the set of shedding vertices is a dominating set. We also develop new classes of well-covered graphs G with the property that G is vertex decomposable and the set of shedding vertices of G is not a dominating set.

Title: Brain networks, brain disorders: Significance of signed graphs
Speaker: Kris Vasudevan, University of Calgary
Co-authors: Gordon Campbell Teskey (Hotchkiss Brain Institute), Paolo Federico (Hotchkiss Brain Institute), and Signe Bray (Radiology Department) of the University of Calgary
Abstract: Graph theory as applied to complex networks and harmonic analysis play a significant role in developing good mathematical models to describe the organization of the brain and to understand the characteristics of brain disorders such as epileptic seizures. The three research projects we are involved in are: (1) Characteristics of epileptic seizures (2) Pre- and post-ictal hippocampal depth recordings (3) Resting state behaviour of brains in children and adolescents. They apply data-driven methods tied to complex network theory and harmonic analysis. I will use the intracranial electroencephalogram (iEEG) data for the first two and the blood oxygenation level dependent functional magnetic resonance (BOLD fMRI) imaging data for the third. In this presentation, I will demonstrate how data-driven methods describe the seizure characteristics at different stages of the ictal period in human focal seizures and in seizure-induced hypoxia in rodents. Also, I will focus on the role of signed graphs in community structures and neuronal dynamics at different spatio-temporal scales.

Title: Combinatorial Aspects of Quantum walks
Speaker: Hanmeng Zhan, University of Waterloo
Abstract: The transition matrix of a quantum walk, either continuous or discrete, is determined by some structure associated with a graph. In this talk, we will discuss some interesting phenomenon of quantum walks, with emphasis on the combinatorial aspects that come into play.

Conference Participants

Kristaps J. Balodis	University of Winnipeg
Jane Breen	University of Manitoba
Richard Brewster	Thompson River University
Steve Butler	Iowa State University
Ada Chan	York University
Rob Craigen	University of Manitoba
Rebecca Danos	University of Winnipeg
Louis Deaett	Quinnipiac University
Serhii Dovhyi	University of Manitoba
Chris Duffy	University of Saskatchewan
Cathi-Lyn Dyck	Brandon University
Aras Erzurumluoğlu	University of Ottawa
Colin Garnett	Black Hills State University
Shonda Gosselin	University of Winnipeg
Allen Herman	University of Regina
Thomas Humphries	Brandon University
Waweva Kaviuki	University of Manitoba
Matt Kroeker	University of Winnipeg
Gary MacGillivray	University of Victoria
Karen Meagher	University of Regina
Brett Meggison	University of Manitoba
Elizabeth McKenzie-Case	Thompson Rivers University
Darian Mclaren	Brandon University
Lucas Mol	University of Winnipeg
Keivan Hassani Monfared	University of Calgary
Joy Morris	University of Lethbridge
Kyrylo Muliarchyk	University of Manitoba
Shahla Nasserasr	Brandon University
Sarah Plosker	Brandon University
Hunter Plowman	Brandon University
Rachel Quinlan	National University of Ireland
Mahsa N. Shirazi	University of Regina
Gurmail Singh	University of Regina
Kevin Vander Meulen	Redeemer University College
Kris Vasudevan	University of Calgary
Harmony (Hanmeng) Zhan	University of Waterloo