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RESEARCH CONNECTION

Sketching the shape of space and time

By Jeff Williams, Ph.D.

Why this research is important

The null surfaces that arise in the *null-surface formulation of general relativity* are central to both Einstein's geometric approach to gravity—where null surfaces represent the event horizons of black holes—and to Cartan's geometric approach to differential equations—where such surfaces are directly related to the existence and determination of solutions of differential equations in general.

How the research was conducted

Children in the 1960s amused themselves with *Etch A Sketch*, a toy that comprised a small grey screen and two knobs that could steer a black dot and trace out a curve whose contortions were limited only by one's imagination. The dot could be moved vertically (y-direction), and horizontally (x-direction), producing a trail which, to a mathematician, represents the graph of a function in the form 'y = something, and x = something else.'

What you need to know

Élie Cartan was born in 1869 in the village of Dolomieu in southeastern France. The son of the local blacksmith, his mathematical ability was recognized soon after he started school. After his studies in Paris, he became a professor at the *Université de Paris-Sorbonne* and established himself as one of the preeminent mathematicians of Europe. In 1938, he proposed a geometric approach aimed at creating a classification system—a taxonomy—for the differential equations of calculus. His system depended on finding solutions to an equation, which—because of its exceptional complexity—proved intractable.

My colleague, Dr. Tina Harriott from Mount Saint Vincent University, and I are working on the *null-surface formulation of general relativity*, which provides a new framework for Einstein's relativistic theory of gravity. In low dimensions, to be precise: *two* dimensions of space and *one* dimension of time, the main equation of the null-surface formulation turns out to be identical to Cartan's remarkable equation of 1938.

In our search to find solutions to our main equation, we were open to the idea that the functions representing such solutions might appear in a variety of forms, including the form that is referred to above. Considerable simplification was achieved by our requiring spacetime to be of dimension

2+1, i.e., two dimensions of space and one of time. We also made the unorthodox assumption that some of our major dependent 'variables' did not vary at all but were constant, or even zero. At that point, we began our search.

What the researchers found

We found the first nontrivial solutions of Élie Cartan's equation—an equation that had remained unsolved for eighty years. These solutions, which are automatically also solutions for the null-surface formulation, are published in the journal *General Relativity and Gravitation*. (See the publications listed below).

The second solution (2019) can be given in the manner 'y = something, and x = something else' that is described above. It uses the familiar log function and a parameter t, which can be interpreted as the time that passes when using some cosmic *Etch A Sketch* device to draw a trajectory across a spacetime of 2+1 dimensions. The solution is written:

$$y = t \div [\, t + \log\sqrt{(1-t)} - \log\sqrt{(1+t)} + 1 \,],\text{ and}$$

$$x = 1 \div [\, t + \log\sqrt{(1-t)} - \log\sqrt{(1+t)} + 1 \,].$$

This solution is interesting because of its simplicity and because the curvature (usually denoted by R) of spacetime varies from point to point and depends in a simple way on t:

$$R = (-1 + 1 \div t^2)^4.$$

This expression involves division by t. When t is zero, the curvature R will become infinite and produce a singularity in spacetime—as would occur at the centre of a black hole or the instant of the 'Big Bang,' when the universe began.

How this research can be used

Our solutions will provide insights into the theory of differential equations (through the relationship with the work of Élie Cartan) and into the theory of gravity (through the relationship with Einstein's general relativity).

About the researchers

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Null-surface formulation, gravity, general relativity, Cartan

Publications based on this research

Harriott, T. A., & Williams, J. G. (2018). Three-variable solution in the (2+1)-dimensional null-surface formulation. *General Relativity and Gravitation*, *50*, 39.

Harriott, T. A., & Williams, J. G. (2019). Petrov type-N solution for the null-surface formulation in 2+1 dimensions. *General Relativity and Gravitation*, *51*, 98.

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