Western Canada Linear Algebra Meeting
Programme

Brandon University

May 29-30, 2021

Organising Committee

Shaun Fallat, Hadi Kharaghani, Steve Kirkland, Sarah Plosker, Michael Tsatsomeros, Pauline van den Driessche

Local Organisers:
Steve Kirkland, Sarah Plosker

The organisers gratefully acknowledge the support of the International Linear Algebra Society (ILAS), the Pacific Institute for the Mathematical Sciences (PIMS), and Brandon University.

Invited Speakers

Ada Chan, York University
Doug Farenick, University of Regina
Judi McDonald, Washington State University

Location

Zoom. Links to be emailed out to all registered participants.

The Western Canada Linear Alegbra Meeting (WCLAM)

Hosted Virtually by:
Brandon University

See https://www.brandonu.ca/wclam/ for more details

	Saturday May 29		Sunday May 30	
TIME	**Zoom 1**	**Zoom 2**	**Zoom 1**	**Zoom 2**
9:15-9:30	Opening Remarks			
9:30-10:30	Judi		Ada	
10:30-10:45	*Break*		*Break*	
10:45-11:15	Louis	Faith	Sooyeong	Kevin
11:15-11:45	Michael C.	Amy	Whitney	Tracy
11:45-1:15	*Lunch Breakout Rooms*		*Lunch Breakout Rooms*	
1:15-1:45	Douglas		Amrita	Ahmad
1:45-2:15	Douglas (cont)		Ayla	Marija
2:15-2:30	*Break*		*Break*	
2:30-3:00	Thomas	Abdul	Christina	Michael K.
3:00-3:30	Lei	Emily	Avleen	Mashael
3:30-3:45	*Break*		*Break*	
3:45-4:15	Colin	Darian	Hermie	Kris
4:15-4:45	Rachel	Jeff	Nathaniel	Junquan
4:45-5:15			Tin-Yau	Ariane
5:15-5:30			Farewell	

Schedule Details

Saturday, May 29

Zoom 1 Chair: Sarah Plosker

9:15-9:30 **Chenkuan Li** (Department Chair), **Bernadette Ardelli** (Dean of Science) Opening Remarks

9:30-10:30 **Judi McDonald** (invited): Orthogonality and Finite Fields

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Zoom 1 Chair: Pauline van den Driessche

10:45-11:15 **Louis Deaett**: The minimum rank problem for patterns and graphs

11:15-11:45 **Michael Cavers**: Potentially stable sign patterns and their digraphs

Zoom 2 Chair: Hadi Kharaghani

10:45-11:15 **Faith Zhang**: Rank one perturbation with a generalized eigenvector

11:15-11:45 **Amy Yielding**: An Exploration into Inertia Sets of Semicliqued Graphs

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Zoom 1

11:45-1:15 Lunch discussions over Zoom breakout rooms

Zoom 1 Chair: Steve Kirkland

1:15-2:15 **Douglas Farenick** (invited): The space of the Toeplitz matrices as an operator system dual

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Zoom 1 Chair: Steve Kirkland

2:30-3:00 **Thomas R. Cameron**: On digraphs with polygonal restricted numerical range

3:00-3:30 **Lei Cao**: Pattern Avoiding (0, 1)-Matrices

Zoom 2 Chair: Michael Tsatsomeros

2:30-3:00 **Abdul Hamid Ganie**: Some new structure of sequence spaces using infinite matrices

3:00-3:30 **Emily Hoopes-Boyd**: The Images of Polynomials Evaluated over Matrices

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Zoom 1 Chair: Shaun Fallat

3:45-4:15 **Colin Garnett**: Edge Turncoat Graphs: The good, the bad and the ugly

4:15-4:45 **Rachel Quinlan**: Alternating sign matrices of finite multiplicative order

Zoom 2 Chair: Sarah Plosker

3:45-4:15 **Darian Mclaren**: Extreme points and bases for stochastic matrices

4:15-4:45 **Jeffrey Stuart**: A Generalization of an Obscure Class of Matrices

Sunday, May 30

Zoom 1 Chair: Sarah Plosker

9:30-10:30 **Ada Chan** (invited): Pretty Good Fractional Revival

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Zoom 1 Chair: Steve Kirkland

10:45-11:15 **Sooyeong Kim**: Equidistant switched hypercubes: their properties and sensitivity analysis under PST

11:15-11:45 **Whitney Drazen**: Fractionally Cospectral Sets on Path Graphs

Zoom 2 Chair: Hadi Kharaghani

10:45-11:15 **Kevin Limanta**: An algebraic interpretation of the (circular) super Catalan numbers

11:15-11:45 **H. Tracy Hall**: Inverse eigenvalues via computer algebra

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Zoom 1

11:45-1:15 Lunch discussions over Zoom breakout rooms

Zoom 1 Chair: Hadi Kharaghani

1:15-1:45 **Amrita Mandal**: Orthogonal matrices that are linear sum of permutation matrices and its applications to quantum walks

1:45-2:15 **Ayla Rodriguez**: On Some Algebro-Geometric Observations of Classical Observables

Zoom 2 Chair: Shaun Fallat

1:15-1:45 **Seyed Ahmad Mojallal**: The minimum number of distinct eigenvalues of threshold graphs

1:45-2:15 **Marija Dodig**: Rank restrictions in Completion Problems

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Zoom 1 Chair: Pauline van den Driessche

2:30-3:00 **Christina Pospisil**: Generalization Theory of Linear Algebra III

3:00-3:30 **Avleen Kaur**: A space-time spectral method for the Stokes problem

Zoom 2 Chair: Michael Tsatsomeros

2:30-3:00 **Michael Kasigwa**: $Z_{v,K}$ and $M_{v,K}$ Type Operators
3:00-3:30 **Mashael AlBaidani**: On the Minimum Number of Extreme Vectors in the Construction of Irreducible and Polyhedral Cones

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Zoom 1 Chair: Sarah Plosker

3:45-4:15 **Hermie Monterde**: Transition Matrices in Quantum State Transfer
4:15-4:45 **Nathaniel Johnston**:Completely positive completely positive maps (and a resource theory for non-negativity of quantum amplitudes)
4:45:5:15 **Tin-Yau Tam**: Inequalities of block positive definite matrices

Zoom 2 Chair: Steve Kirkland

3:45-4:15 **Kris Vasudevan**: Ideas from Random Matrix Theory to study neuronal correlations in noisy environments: Intracranial EEG recordings of the CA1 of the hippocampus
4:15-4:45 **Junquan Xiao**: Coninvolutory matrices, multi-affine polynomials, and invariant circles
4:45-5:15 **Ariane Masuda**: A Conjugate Directions-Type Procedure for Quadratic Multiobjective Optimization

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Zoom 1

5:15-5:30 Farewell ☹

Abstracts (alphabetical by speaker)

Title On the Minimum Number of Extreme Vectors in the Construction of Irreducible and Polyhedral Cones

Speaker Mashael AlBaidani,
mashaelalbaidani@gmail.com

Abstract In this talk, we explore the relationship between the minimum number of extreme vectors in a cone and the spectrum of a matrix for which the cone is invariant. Moreover, we present some results include identifying properties of the extremal vectors of the cone based on the peripheral spectrum of the associated matrix, thus contributing to a lower bound for the number of extreme vectors the cone must contain, as well as techniques for creating eventually invariant proper cones, with a small number of extreme vectors, based on the spectral properties of the given matrix.

Title On digraphs with polygonal restricted numerical range

Speaker Thomas R. Cameron (trc5475@psu.edu)
Department of Mathematics, Penn State Behrend, Erie, PA, USA

Co-authors Tracy Hall (Hall Labs, LLC), Ben Small, and Alex Wiedemann (Davidson College)

Abstract

Spectral graph theory has a long and successful history of characterizing graphs. However, there has been far less success in the study of the spectra of directed graphs, which is mainly due to the asymmetry of the associated matrices. In 2020, Cameron et al. introduced the concept of the *restricted numerical range* of a digraph, which can be used to characterize digraphs and study their algebraic connectivity. The authors were able to characterize digraphs whose restricted numerical range was equal to a single point or a horizontal or vertical line segment.

In this talk, we investigate digraphs whose restricted numerical range is polygonal, i.e., a convex polygon in the complex plane, which includes the previously mentioned degenerate polygons as a special case. In particular, we provide computational methods for identifying these *polygonal digraphs*, and show that these digraphs can be broken into three disjoint classes: *normal*, *restricted-normal*, and *pseudo-normal* digraphs. Each class is closed under the graph complement, and we prove sufficient conditions for the structure of digraphs in each class. Finally, we prove that the sufficient conditions for restricted-normal digraphs are also necessary when the digraph's order is square-free.

Title Pattern Avoiding (0, 1)-Matrices

Speaker Lei Cao, Nova Southeastern University (lcao@nova.edu)

Co-authors Richard Brualdi, University of Wisconsin - Madison

Abstract

We investigate pattern-avoiding $(0,1)$-matrices as generalizations of pattern-avoiding permutations. Our emphasis is on 123-avoiding and 312-avoiding patterns for which we obtain exact results as to the maximum number of 1's such matrices can have. We also give algorithms which, when carried out in all possible ways, construct all of the pattern-avoiding matrices of these two types.

Title Potentially stable sign patterns and their digraphs

Speaker Michael Cavers, University of Calgary (mcavers@ucalgary.ca)

Abstract

A sign pattern $\mathcal{A}$ is a matrix with entries in $\{+,-,0\}$. If there exists a real stable matrix with pattern $\mathcal{A}$, i.e., a matrix with each of its eigenvalues having negative real part, then $\mathcal{A}$ is potentially stable. First, necessary conditions for a polynomial to be stable are interpretted in the context of digraphs. For example, if the digraph of $\mathcal{A}$ has exactly one loop, no 3-cycles and every 2-cycle vertex disjoint from the loop, then $\mathcal{A}$ is not potentially stable. Second, known bounds on the minimum number of nonzero entries in an irreducible potentially stable sign pattern are discussed and extended. A conjecture is stated with supporting evidence. Finally, potentially stable nonzero patterns (matrices with entries in $\{*,0\}$) of order at most 4 are characterized.

Title Pretty Good Fractional Revival

Speaker Ada Chan, York University (ssachan@yorku.ca)

Co-authors
Whitney Drazen, Northeastern University;
Or Eisenberg, Harvard University;
Mark Kempton, Brigham Young University;
Gabor Lippner, Northeastern University.

Abstract

Let X be a graph with adjacency matrix A. The transition matrix of the continuous-time quantum walk is e^{-itA}. We say that fractional revival occurs between vertices a and b in X at time τ if

$$e^{-i\tau A} = \begin{bmatrix} H_1 & 0 \\ 0 & H_2 \end{bmatrix}, \quad (1)$$

for some non-diagonal 2×2 matrix H_1 with its rows and columns indexed by a and b. Fractional revival is relevant in entanglement generation but graphs with fractional revival are rare. For instance, only paths with length at most four have fractional revival.

In this talk, we introduce a relaxation called pretty good fractional revival. We show that if pretty good fractional revival occurs between a and b in X then the adjacency algebra $\langle A \rangle$ contains a matrix of the block diagonal form described in (1). We give the spectral properties of the graphs that satisfy this necessary condition, and use them to characterize the paths that have pretty good fractional revival.

Title The minimum rank problem for patterns and graphs

Speaker Louis Deaett, Quinnipiac University (louis.deaett@quinnipiac.edu)

Co-authors Derek Young, Mount Holyoke College

Abstract

Replacing each nonzero entry of a matrix with the symbol $*$ gives the zero-nonzero pattern of the matrix. What can this combinatorial description tell us about the rank of a matrix? This question boils down to determining the smallest possible rank given such a description. There is a related problem for graphs, namely to determine the smallest rank of a symmetric matrix whose off-diagonal zero-nonzero pattern matrices that of the adjacency matrix of the graph. This is known as the minimum rank of the graph. The goal of this talk is to discuss known connections between these two problems, and current work in the directions of strengthening these connections.

Title Rank restrictions in Completion Problems

Speaker Marija Dodig, Department of Mahematics, University of Lisbon; and MISANU Belgrade (msdodig@fc.ul.pt)

Co-authors Marko Stosic, Department of Mahematics, IST, University of Lisbon; and MISANU Belgrade

Abstract

In this talk we shall consider rank restriction in Matrix Pencil Completion Problems. In particular, we will present novel results on double and single rank restrictions. These results directly generalize all of the existing results on Matrix Pencil Completion Problems existing in the literature.

For example, as a corollary of the results on single rank restrictions we shall describe the possible Kronecker invariants of a matrix pencil with a prescribed a quasi-regular subpencil.

Also, we shall show the dual result - we describe the possible Kronecker invariants of a quasi-regular matrix pencil with a prescribed subpencil, without any kind of additional restrictions.

Title Fractionally Cospectral Sets on Path Graphs

Speaker Whitney Drazen, Northeastern University (drazen.w@northeastern.edu)

Abstract

Two vertices u and v in a graph G are cospectral if the subgraphs $G\backslash u$ and $G\backslash v$ formed by deleting vertices u and v respectively have the same characteristic polynomials. Equivalently, given the adjacency matrix A of a graph, we say that u and v are cospectral if $(A^k)_{u,u} = (A^k)_{v,v}$ for all nonnegative integers k. This concept is essential to the study of perfect state transfer. Arising from the more general state transfer phenomena called fractional revival, there is an analogous relation between vertices called fractional cospectrality. We say u and v are fractionally cospectral if there is a real number c such that $(A^k)_{u,u} = (A^k)_{v,v} + c(A^k)_{u,v}$ for all nonnegative integers k. In this talk we explain how to extend this notion to larger subsets and explore families of fractionally cospectral vertices on path graphs. These families will include sets of arbitrarily large size.

Title The space of the Toeplitz matrices as an operator system dual

Speaker Douglas Farenick, University of Regina (douglas.farenick@uregina.ca)

Abstract A recent paper of A. Connes and W.D. van Suijlekom identifies the operator system of $n \times n$ Toeplitz matrices with the dual of the space of all trigonometric polynomials of degree less than n. This lecture examines this identification in somewhat more detail, showing that the Connes–van Suijlekom isomorphism is in fact a unital complete order isomorphism of operator systems. This complete order isomorphism yields new insights into the positivity of block Toeplitz matrices, which are viewed herein as elements of tensor product spaces of an arbitrary operator system with the operator system of $n \times n$ complex Toeplitz matrices. In particular, it is shown that min and max positivity are distinct if the blocks themselves are Toeplitz matrices, and that the maximally entangled Toeplitz matrix ξ_n is entangled in the operator system all continuous $n \times n$ Toeplitz-matrix valued functions f on the unit circle S^1 whose Fourier coefficients $\hat{f}(k)$ vanish for $|k| \geq n$, but is approximately separable in the operator system all continuous $n \times n$ Toeplitz-matrix valued functions f on S^1.

Title Some new structure of sequence spaces using infinite matrices

Speaker Abdul Hamid Ganie,
Department of Applied Science and Humanities,
SSM College of Engineering and Technology Pattan, Kashmir
(ashamidg@rediffmail.com)

Abstract Quite recently, the basic structure of Lucas sequences were introduced and studied by Karakas and Karabudak. The basic structure of this paper is to introduce some new analysis of spaces and Lucas matrix using modulus function with Lucas numbers. In it we further study linear isomorphism property and computation of BK-property will be carried. We further look some basic inclusion property concerning the given results.

Title Edge Turncoat Graphs: The good, the bad and the ugly

Speaker Colin Garnett, Black Hills State University (Colin.Garnett@bhsu.edu)

Co-authors Jeffrey Winter, Black Hills State University, Kerry Tarrant, University of Iowa

Abstract

Cops and robbers is a vertex pursuit game played on a simple graph. The win/loss state for each graph is known and well studied. We ask the question, when does the removal or addition of any edge change this win/loss state? We focus on the case when adding an edge changes the state from robber-win to cop-win. These are called maximally robber-win graphs, while we have not found a characterization of such graphs, we have come up with a variety of examples.

Title Inverse eigenvalues via computer algebra

Speaker H. Tracy Hall, Hall Labs, LLC (h.tracy@gmail.com)

Abstract

The inverse eigenvalue problem for a graph asks which spectra can be obtained for symmetric matrices with a prescribed pattern of nonzero off-diagonal entries. Many questions can be resolved with general techniques for all graphs, or graphs from certain infinite families, but it is sometimes desirable to answer questions for a specific graph by directly parametrizing the corresponding set of matrices. For small enough graphs this can be done by hand at the whiteboard, and for large enough graphs the calculations generally become intractable, but there is an intermediate range in which computer algebra becomes necessary and is often sufficient. Judicious choices in the parametrization can make a difference, and techniques such as Groebner bases can be useful. I will talk about some open questions that I have resolved in the last year using the Sagemath platform: Zero forcing does not always give a tight bound on minimum rank for circulant graphs, and the minimum number of distinct eigenvalues for a disconnected graph is not necessarily the maximum over its components.

Title The Images of Polynomials Evaluated over Matrices

Speaker Emily Hoopes-Boyd, Kent State University (ehoopes1@kent.edu)

Co-authors Aria Beaúpre, Harvey Mudd College; Grace O'Brien, The University of Michigan

Abstract

Let $M_n(F)$ be the ring of $n \times n$ matrices over an infinite field, F. The L'vov-Kaplansky conjecture states that the image of a multilinear polynomial evaluated over $M_n(F)$ is a vector space. This statement is still an open problem, but many partial results have been proven within the last decade. We will consider this problem in a slightly different context; rather than taking the matrix entries to be from an infinite field, we will consider matrices over

an algebraically closed skew field, which we will denote by K. We will show that the image of any multilinear polynomial with coefficients from K, evaluated over $M_n(K)$, is $M_n(K)$. We will also prove that any matrix in $M_n(K)$ may be written as the sum of three or fewer elements from the image of any generalized polynomial. In particular, the image of the polynomial $xy - yx$ has some special properties over a variety of matrix rings.

Title Completely positive completely positive maps (and a resource theory for non-negativity of quantum amplitudes)

Speaker Nathaniel Johnston (njohnston@mta.ca)

Department of Mathematics & Computer Science, Mount Allison University, Sackville, NB E4L 1E4, Canada

Co-authors Jamie Sikora, Perimeter Institute for Theoretical Physics, Waterloo, ON, Canada N2L 2Y5

Abstract

We examine quantum states which are non-negative mixtures of pure states with non-negative amplitudes (in a fixed basis) and the channels which preserve them. These states are exactly those that are completely positive (CP), and we show how several standard properties of CP matrices, such as the CP-rank, correspond to physical properties of these states. We also introduce the family of quantum channels that preserve CP states, which we call *completely positive completely positive (CPCP)*, since quantum channels are also (very confusingly) called completely positive. We show that CP quantum states and the CPCP maps that preserve them constitute a (physically well-motivated) quantum resource theory analogous to that of quantum entanglement. Finally, we investigate several ways of measuring how resourceful a state is in this theory (which roughly means how far away it is from being CP).

Title: $Z_{v,K}$ and $M_{v,K}$ Type Operators

Speaker: Michael Kasigwa, The Ohio State University at Newark (kasigwa.1@osu.edu)

Abstract

$Z_{v,K}$ and $M_{v,K}$ are square matrices of the form $A = sI - B$, where $r(B)$ is the spectral radius of the square matrix B, $r(B) \geq s \geq 0$, K is a cone and B is eventually K-invariant. We discuss the theory of $Z_{v,K}$ and $M_{v,K}$-matrices in the context of extending it to $Z_{v,K}$ and $M_{v,K}$- operators. Recently some authors extended the notion of Z and M-matrices to Z and M-operators, respectively. We seek to develop on this further through the following questions. (1) In what ways can generalization of Z and M matrices to $Z_{v,K}$ and $M_{v,K}$-matrices, respectively, carry over to operators in general? (2) Authors of another study on linear differential systems, $\dot{X}(t) = AX(t)$, where A is an n-square matrix, $X_0 = X(0)$ an

n-dimensional vector and $t \geq 0$, characterized solutions that become and remain nonnegative in finite time, what is the analogue of this for eventual invariance of cones in ordered Banach spaces?

Title A space-time spectral method for the Stokes problem

Speaker Avleen Kaur, University of Manitoba (kaura349@myumanitoba.ca)

Co-authors Shaun Lui, University of Manitoba (shaun.lui@umanitoba.ca)

Abstract

In this work, we consider the Stokes equations in steady and unsteady states, along with Dirichlet boundary conditions and an initial condition in the latter case. We impose the $\mathbb{P}_N - \mathbb{P}_{N-2}$ spectral Galerkin scheme in space by using a recombined Legendre polynomial basis resulting in exponential convergence in space. For the unsteady state, we implement spectral collocation in time, thus giving exponential convergence in both space and time. The global spectral operator for both schemes is a saddle point matrix. We prove the 2-norm estimates for every block of the two operator matrices. We proceed to show that the condition number for the global spectral operator for the steady-state scheme is $O(N^4)$, where N is the number of spectral modes in each direction. We have preliminary results on the condition number of the unsteady-state scheme. We also exhibit the numerical results of this scheme applied to the unsteady Navier-Stokes problem.

Title Equidistant switched hypercubes: their properties and sensitivity analysis under PST

Speaker Sooyeong Kim, University of Manitoba (kims3428@myumanitoba.ca)

Co-authors Steve Kirkland, University of Manitoba

Abstract Our ultimate goal of this work is to switch interactions between qubits in a quantum spin network corresponding to a hypercube, in order for the manipulated spin network to become insensitive to external environments under perfect state transfer (PST). We present differences and similarities between hypercubes and the resulting graphs (equidistant switched hypercubes) regarding the graph structure, PST, and the sensitivity of PST. We provide numerical results about our goal, and finally propose a related conjecture.

Title An algebraic interpretation of the (circular) super Catalan numbers

Speaker Kevin Limanta, UNSW Sydney (k.limanta@unsw.edu.au)

Co-author Norman Wildberger, UNSW Sydney (n.wildberger@unsw.edu.au)

Abstract

In this talk, I will give a brief introduction of the **super Catalan numbers**

$$S(m,n) = \frac{(2m)!(2n)!}{m!n!(m+n)!}$$

and their curious connection to the problem of integrating polynomials over various unit circles in different geometries over finite field of characteristic not two via an intermediary family of numbers, called the **circular super Catalan numbers**

$$\mathrm{CS}(m,n) = \frac{S(m,n)}{4^{m+n}}.$$

More specifically, we consider three unit circles of particular interest, the standard Euclidean unit circle and two Minkowski-Einstein unit circles

$$\begin{aligned} S_{b,q} &= \{(x,y) \in \mathbf{F}_q :\ x^2+y^2=1\}, \\ S_{r,q} &= \{(x,y) \in \mathbf{F}_q :\ x^2-y^2=1\}, \\ S_{g,q} &= \{(x,y) \in \mathbf{F}_q :\ xy=1\} \end{aligned}$$

where $\mathbf{F}_q$ is *the* finite field of size $q = p^k$ where $p > 2$ and we will see that not only does this approach give a generalisation of the existing theory over the Euclidean geometry, the three geometries are interconnected.

We will also see that this gives us the technology to do (finite field) Fourier analysis in a completely algebraic way.

Title Orthogonal matrices that are linear sum of permutation matrices and its applications to quantum walks

Speaker Amrita Mandal, Indian Institute of Technology Kharagpur (amritamandal@iitkgp.ac.in)

Co-authors Rohit S. Sarkar and Bibhas Adhikari,
Department of Mathematics, Indian Institute of Technology Kharagpur

Abstract

A permutative matrix is defined as a matrix each row of whose is a permutation of any other row. Thus permutative matrices can be thought as a generalization of permutation matrices. In this talk, we focus on the following question: Which permutative matrices are orthogonal?

We show that any orthogonal matrix, which is a linear sum of permutation matrices, is always permutative when the matrices are of order 3×3 and the set of all such matrices form

a matrix group. On the other hand, we show that this is no longer true for 4×4 permutative orthogonal matrices. In this context, we revisit the open problem: under what condition a linear sum of permutation matrices is orthogonal? We provide a complete solution of this problem for 3×3 matrices by characterizing all permutative matrices of order 3×3 that are orthogonal. Further we discuss examples of orthogonal 4×4 matrices that are linear sum of permutation matrices and non-permutative. The examples of classes of such matrices are derived using the concept of combinatorial orthogonality of a $(0, 1)$ matrix corresponding to the pattern of zero-nonzero entries of a matrix

Finally, we propose permutative orthogonal matrices as a generalization of the Grover matrix. Then we define discrete-time coined quantum walks on cycles and two-dimensional lattices such that the coin operators are permutative orthogonal matrices, and we analyze periodicity and localization phenomena of such quantum walks.

Title A Conjugate Directions-Type Procedure for Quadratic Multiobjective Optimization

Speaker Ariane Masuda, New York City College of Technology (CUNY) (amasuda@citytech.cuny.edu)

Co-authors Ellen H. Fukuda (Kyoto University) and L. M. Graña Drummond (Universidade Federal do Rio de Janeiro)

Abstract

We propose an extension of the real-valued conjugate directions method for unconstrained quadratic multiobjective problems. As in the single-valued counterpart, the procedure requires a set of directions that are simultaneously conjugate with respect to the positive definite matrices of all quadratic objective components. Likewise, the multicriteria version computes the steplength by means of the unconstrained minimization of a single-variable strongly convex function at each iteration. When it is implemented with a weakly-increasing (strongly-increasing) auxiliary function, the scheme produces weak Pareto (Pareto) optima in finitely many iterations.

Title Orthogonality and Finite Fields

Speaker Judi McDonald, Washington State University (jmcdonald1@wsu.edu)

Co-authors Aishah Basha, Washington State University

Abstract Orthogonality has been a useful tool in both theoretical and numerical techniques used to analyze matrices and vectors over the real and complex numbers. Analogous to the idea of a complex conjugate, we can define the conjugate of a element in the finite field $\mathbb{F}_{q^m}$ over the field $\mathbb{F}_q$ and set up a bilinear form $\langle\ ,\ \rangle : {\mathbb{F}_{q^m}}^n \to {\mathbb{F}_{q^m}}^n$. Using this bilinear form, we define $\langle x, y\rangle = x^{\triangle} y$, where $x^{\triangle}$ represents the conjugate transpose of x. In my talk I will discuss properties of orthogonal bases, numerical ranges, unitary, Hermitian, and normal matrices under this notion of orthogonality.

Title Extreme points and bases for stochastic matrices

Speaker Darian Mclaren, University of Waterloo (da2mclar@uwaterloo.ca)

Co-authors
Sarah Plosker, Brandon University
Lei Cao, Halmos College, Nova Southeastern University

Abstract

Birkhoff's theorem states that the set of $n \times n$ doubly stochastic matrices is the convex hull of the $n \times n$ permutation matrices. Thereby giving a characterization of the extreme points. We relax the condition of doubly stochastic to just stochastic (rows sum to 1) to provide a characterization of the extreme points and a construction of a basis for the following convex sets: the set of $m \times n$ stochastic matrices $\Gamma_{m,n}$; and the set of $m \times n$ centrosymmetric stochastic matrices $\Gamma^{\pi}_{m,n}$.

Title The minimum number of distinct eigenvalues of threshold graphs

Speaker Seyed Ahmad Mojallal, University of Regina
(mojallal.seyed.ahmad@uregina.ca)

Co-author Shaun Fallat, University of Regina

Abstract

For a graph G, we associate a family of real symmetric matrices, $S(G)$, where for any $M \in S(G)$, the location of the nonzero off-diagonal entries of M are governed by the adjacency structure of G. The minimum number of distinct eigenvalues, taken over $S(G)$ compatible with the graph G, is denoted by $q(G)$.

Threshold graphs can be characterized in many ways. One way of obtaining a threshold graph is through an iterative process which starts with an isolated vertex, and where, at each step, either a new isolated vertex is added, or a vertex adjacent to all previous vertices (dominating vertex) is added.

In this talk, we study the spectral invariant of $q(G)$ for connected threshold graphs of a fix order n.

Title: Transition Matrices in Quantum State Transfer

Presenter: Hermie Monterde (monterdh@myumanitoba.ca), University of Manitoba, Winnipeg, Manitoba, Canada

Supervisors: Stephen Kirkland (University of Manitoba) and Sarah Plosker (Brandon University)

Abstract. A quantum spin network can be modelled using an undirected graph G. By axioms of quantum mechanics, the transfer of state from one vertex to another is governed

by the matrix $U(t) = e^{itM}$, where M is a matrix associated to the graph G depending on the dynamics. The matrix $U(t)$ is called the transition matrix of X, and as it happens, the entries of $U(t)$ give information about the probability of state transfer between any two vertices of G at time t. In this talk, we take a linear algebraic approach to analyzing the basic properties of $U(t)$, and examine the elementary properties of various types of state transfer in relation to $U(t)$.

Title Generalization Theory of Linear Algebra III

Speaker Christina Pospisil (pospisil.christina@gmx.de)

Abstract

This talk continues the presentations Generalization Theory of Linear Algebra I+II from the Joint Mathematics Meeting (JMM) 2019 and JMM 2020. In the first part an algorithm for multiplying and adding matrices regardless of dimensions via an embedding and inverses for non-injective mappings in one dimension were presented (first part was presented at JMM 2019). The second part presented inverses for non-injective mappings in multiple dimensions, inverses for non-surjective mappings in one and multiple dimensions and introduced a general determinant theory (second part was presented at JMM 2020). The third part is dedicated to a further generalization regarding tensors with first applications in physics. In future work there will be further operations and applications to physics and other natural sciences be explored.

Title Alternating sign matrices of finite multiplicative order

Speaker Rachel Quinlan, National University of Ireland, Galway (rachel.quinlan@nuigalway.ie)

Co-authors Cian O'Brien, NUI Galway

Abstract An alternating sign matrix (ASM) is a square $(0, 1, -1)$-matrix in which the non-zero entries alternate in each row and column, beginning and ending with 1. Examples of ASMs include permutation matrices, and there are contexts in which the set of $n \times n$ ASMs may be seen as a natural extension or completion of the set of permutation matrices. Unlike the permutation matrices which form a group, the ASMs are not equipped with any apparent algebraic structure, and the permutation matrices are the only ones to generate cyclic groups whose elements are all ASMs. Nevertheless, there exist (non-permutation) $n \times n$ ASMs that have finite multiplicative order, and that have have finite orders not occurring in the symmetric group of degree n. We present constructions of some such matrices, and suggest some questions.

Title On Some Algebro-Geometric Observations of Classical Observables

Speaker Ayla Rodriguez, Black Hills State University (ayla.rodriguez@yellowjackets.bhsu.edu)

Co-authors Dr. Parthasarathi Nag, Black Hills State University (p.nag@bhsu.edu)

Abstract The set of observables $\mathcal{O}$ of a classical system are continuous real-valued functions on the phase space T^*Q where Q is the configuration space, which can be characterized by the self-adjoint elements of a separable commutative unital C^* algebra $\mathcal{A}$. The Gelfand Transform establishes an isometric isomorphism of the $\mathcal{A}$ onto the C^* algebra of continuous complex-valued functions on the spectrum of $\mathcal{A}$, denoted $spm(\mathcal{A})$, called the Gelfand spectrum.

In this presentation, we shall discuss some algebro-geometric properties of the classical observables characterized by the Gelfand spectrum.

Title A Generalization of an Obscure Class of Matrices

Speaker Jeffrey Stuart, Pacific Lutheran University (stuartjl@plu.edu)

Co-authors No one who will admit to it

Abstract

There is perennial interest in generalizations of circulant matrices and k-potent matrices. Recent papers by Minerva Catral, Leila Lebtahi, Nestor Thome, myself and others have investigated the $\{R, s+1, k\}$-potent matrices. These are matrices A such that $RA^{k+1} = AR$ where R is a k-involutory matrix. This talk discusses what happens when these conditions are generalized to $RA^{k+p} = A^pR$ for some integer $p > 1$.

Title: Inequalities of block positive definite matrices

Speaker: Tin-Yau Tam, University of Nevada, Reno, Nevada, USA, ttam@unr.edu

Abstract: We will first talk about the classical Fischer's inequality for positive semidefinite (psd) matrices and then we give its generalization in the context of completely positive maps. Partial traces are introduced. Extensions of several existing inequalities on the determinants of partial traces are obtained. We improve a determinantal inequality given by M. Lin. We also introduce partial transpose of a block psd matrix, partial positive transpose (PPT) matrices, and then obtain some related results. Finally we discuss geometric mean of psd matrices and some open questions on singular values inequalities.

Title Ideas from Random Matrix Theory to study neuronal correlations in noisy environments: Intracranial EEG recordings of the CA1 of the hippocampus

Speaker Kris Vasudevan, University of Calgary (vasudeva@ucalgary.ca)

Co-authors Michael Cavers, University of Calgary

Abstract

Random Matrix Theory is frequently visited in the areas of multivariate statistical analysis, mathematical physics, wireless communication, quantum gravity, edge-weighted random graphs, condensed matter physics, financial correlation matrices, and in biology. The numerical evaluation of the data to seek specific numbers such as moments, quantiles, or correlations requires a clear knowledge of the underlying probability distributions. Since the higher dimensionality and large sizes of the data pose questions about the extreme eigenvalues of their covariance matrices, the significance of classical probability distributions is questioned.

In this presentation, we focus on neuronal correlation matrices computed from intracranial EEG data using a specific data example from the CA1 region of the hippocampus; the CA1 region plays a significant role in understanding the memory and learning functions of the brain. We examine a method to extract the nonrandom properties from these empirical correlation matrices and illustrate how spectral graph theory ideas may be used to analyze the community structure inherent in corresponding networks.

Title Coninvolutory matrices, multi-affine polynomials, and invariant circles

Speaker Junquan Xiao, Department of Statistical and Actuarial Sciences, Western University (jxiao48@uwo.ca)

Co-authors Hristo Sendov, Department of Statistical and Actuarial Sciences, Western University

Abstract

We investigate a connection between certain (skew-)coninvolutory matrices, multi-affine polynomials and n-tuples of circles in the complex plane, that are invariant under such polynomials. These matrices were investigated by Roger Horn and his student Denis Merino among others.

It is well-known that any non-degenerate Möbius transformation $T(z) = (az+b)/(cz+d)$, sends circles into circles. If for any $z_1 \in \mathbb{C}^*$, the extended complex plane, one lets $z_2 := T(z_1)$, then trivially the pair (z_1, z_2) is a solution of the bi-affine polynomial $p(z_1, z_2) := cz_1z_2 - az_1 + dz_2 - b$. We investigate a natural generalization of these observations and consider a multi-affine polynomial $p(z_1, \ldots, z_n)$ of degree n. We say that the n-tuple of circles $(C_1, \ldots, C_n)$ in $\mathbb{C}^*$ are *invariant* under p, if for any $k \in \{1, \ldots, n\}$, and any $z_i \in C_i, i \neq k$, there exists a $z_k \in C_k$, such that $(z_1, \ldots, z_n)$ is a solution of p. Given an n-tuple of circles $(C_1, \ldots, C_n)$, we give two characterizations of all multi-affine polynomials that preserve them. The opposite problem: given a multi-affine polynomial, find all n-tuples of circles that are invariant under that polynomial, turns out to be much harder. We answer the opposite question only for multi-affine symmetric polynomials.

Title An Exploration into Inertia Sets of Semicliqued Graphs

Speaker Amy Yielding, Eastern Oregon University (ayielding@eou.edu)

Co-authors E. Collins, T.J. Hunt, J.J. Jacobs, J. Juarez, T.A. Rhoton, H.J. Sell

Abstract In this talk we investigate inertia sets of simple connected undirected graphs. The main focus being on the shape of their corresponding inertia tables, in particular whether or not they are trapezoidal. We introduce a special family of graphs, coined semicliqued graphs and denoted $\widetilde{K}G$. We establish the minimum rank and inertia sets of some $\widetilde{K}G$ in relation to the original graph G. We also explore cases of such graphs that are trapezoidal as well ones that resist becoming trapezoidal.

Title Rank one perturbation with a generalized eigenvector

Speaker Faith Zhang, University of Massachusetts (yzhang@math.umass.edu)

Abstract

The relationship between the Jordan structures of two matrices sufficiently close has been largely studied in the literature, among which a square matrix A and its rank one updated matrix of the form $A + xb^*$ are of special interest. The eigenvalues of $A + xb^*$, where x is an eigenvector of A and b is an arbitrary vector, were first expressed in terms of eigenvalues of A by Brauer in 1952. Jordan structures of A and $A + xb^*$ have been studied, and similar results were obtained when a generalized eigenvector of A was used instead of an eigenvector. However, in the latter case, restrictions on b were placed so that the spectrum of the updated matrix is the same as that of A. There do not seem to be results on the eigenvalues and generalized eigenvectors of $A + xb^*$ when x is a generalized eigenvector and b is an arbitrary vector. In this paper we show that the generalized eigenvectors of the updated matrix can be written in terms of those of A when a generalized eigenvector of A and an arbitrary vector b are involved in the perturbation.

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